International Journal of Modern Physics A,

THE ENDPOINT REGION IN RADIATIVE QUARKONIA DECAYS[a,b]

F. HAUTMANN

Department of Physics, Pennsylvania State University, University Park PA 16802

We consider the inclusive radiative decays *quarkonium* $\to \gamma +$ *hadrons* and examine the effects of soft QCD radiation on the photon energy spectrum near the endpoint.

The use of perturbation theory to describe radiative decays of quarkonia is based on the fact that, as long as the velocity v of the heavy quark is small, these decay processes involve two widely separated distance scales: the scale $1/(mv^2)$ over which the quark and antiquark bind into the quarkonium and the scale $1/m$ over which the quark-antiquark pair decays (with m the heavy quark mass). By expanding about the nonrelativistic limit $v \to 0$, one may treat the process as the product of a long-distance, nonperturbative factor, containing all of the bound-state dynamics, and a short-distance factor, describing the annihilation of the heavy quark pair and computable as a power series expansion in α_s.

However, near the exclusive boundary of the phase space, where the photon's energy E_γ approaches its kinematic limit, both the expansion to fixed order in α_s and the expansion to fixed order in v become inadequate to represent correctly the physics of the decay. On one hand, potentially large terms in $\ln(1-z)$ (with $z = E_\gamma/m$) appear in the coefficients of the expansion in α_s to all orders[1]. On the other hand, classes of relativistic corrections that by power counting are higher order in v become enhanced by powers of $1/\alpha_s$[2,3,4]. These behaviors depend on the soft color radiation associated with the decay. In both cases, reliable results can only be obtained after resummation of the enhanced contributions. An analogous sensitivity to infrared dynamics is seen in calculations that incorporate models for the hadronization of partons[5,6,7]: nonperturbative contributions are found to be essential to describe the photon energy spectrum[8,9] near the endpoint.

In this talk we outline a treatment of the infrared radiation near the kinematic boundary, including soft-gluon coherence effects, and give an argument for the cancellation of all the corrections in $\ln(1-z)$ in the short distance coefficient for the color-singlet Fock state in the quarkonium[10]. We focus on processes in which the photon is directly coupled to the heavy quarks, because near the endpoint these contributions dominate the contributions in which the photon is produced by

[a]Talk at the DPF2000 Meeting, Ohio State University, 9-12 August 2000.
[b]Work funded in part by the US Department of Energy.

fragmentation[11] of light quarks and gluons.

Consider the lowest order contribution for a $^3S_1$ quarkonium H of mass M in a color singlet state, $H \to \gamma gg$. The normalized photon spectrum at this order is[12]

$$\frac{1}{\Gamma_0}\,\frac{d\Gamma_0}{dz} = \frac{1}{\pi^2-9}\,\int_0^1 dx_1 \int_0^1 dx_2\; \mathcal{M}_0(x_1,x_2,z)\;\delta(z-2+x_1+x_2)\;\Theta(x_1+x_2-1)\;, \tag{1}$$

with

$$\mathcal{M}_0(x_1,x_2,z) = \frac{(1-x_1)^2}{z^2x_2^2} + \frac{(1-x_2)^2}{z^2x_1^2} + \frac{(1-z)^2}{x_1^2x_2^2} \quad . \tag{2}$$

Here x_1, x_2 denote gluon energy fractions, $x_i = 2E_i/M$. The result (1) is well approximated by a spectrum rising linearly with z. It goes to a constant as $z \to 1$.

To higher perturbative orders, logarithmic corrections arise from soft and collinear gluon radiation. By power counting the leading behavior of the spectrum as $z \to 1$ is of the type[1]

$$\frac{1}{\Gamma}\,\frac{d\Gamma}{dz} \sim \text{const.} + \sum_{k=1}^{\infty} c_k\;\alpha_s^k\;\ln^{2k}(1-z)\;, \qquad z \to 1 \quad . \tag{3}$$

The coherent branching algorithm[13] provides a method to evaluate the relevant multiparton matrix elements in the soft and collinear regions. This algorithm effectively replaces the calculation of higher-loop Feynman graphs by the calculation of tree-level graphs, in which the angular phase space is subject to ordering constraints at each branching. The basis for this method is the coherence property of soft gluon emission.

This method has been developed and applied to the study of event-shape observables in e^+e^- annihilation[14]. The photon spectrum in decays of quarkonia can be treated in a similar manner. Following this approach, the photon spectrum can be expressed, to all orders in α_s and to the leading and next-to-leading accuracy in $\ln(1-z)$, in terms of the on-shell amplitude for the tree-level process $H(P) \to \gamma(k)g(k_1)g(k_2)$ and the mass distributions J_g for the time-like jets defined by the branching algorithm[10]:

$$\begin{aligned}\frac{1}{\Gamma}\,\frac{d\Gamma}{dz} \;\; &= \;\; \int \frac{d^4k}{(2\pi)^3}\frac{d^4k_1}{(2\pi)^3}\frac{d^4k_2}{(2\pi)^3} \\ &\times \;\; (2\pi)^4\delta^4(P-k-k_1-k_2)\,\delta\left(z-\frac{2P\cdot k}{M^2}\right)\delta_+(k^2) \\ &\times \;\; \mathcal{M}^{(\text{tree})}(P,k_1,k_2)\; J_g\left((k_1+k_2)^2,k_1^2\right)\; J_g\left((k_1+k_2)^2,k_2^2\right) \qquad . \end{aligned} \tag{4}$$

The precise definition of $J(p^2,k^2)$ is given in ref.[14]. The first argument in J is the coherence scale; the second argument is the jet mass. To lowest order, $J_g(p^2,k^2) = \delta(k^2) + \ldots$. In this case in Eq. (4) we reobtain the phase space for the γgg final state; then the tree-level amplitude $\mathcal{M}^{(\text{tree})}$ becomes proportional to the amplitude $\mathcal{M}_0$ of Eq. (2), and Eq. (4) simply gives back the result (1). In general, J satisfies an evolution equation of the form

$$J(p^2,k^2) = \delta(k^2) + \int \alpha_s(p'^2)\mathcal{K}(p'^2,k^2) \otimes J(p'^2,k^2) \quad , \tag{5}$$

where expressions for the kernel $\mathcal{K}$ are known to leading and next-to-leading order[14]. In the present discussion we limit ourselves to considering a double logarithmic approximation to J. In this approximation[13]

$$\int dk^2 \ J_g(p^2,k^2) \ \Theta(Q^2-k^2) \approx \exp\left[-\frac{\alpha_s}{2\pi} \ C_A \ \ln^2\left(\frac{p^2}{Q^2}\right)\right] \quad . \tag{6}$$

To determine the logarithmic contributions (3) to the photon spectrum, we need to evaluate the branching formula (4) explicitly. We are interested in the angular-ordered, coherent region

$$k_1^2 \ , \ k_2^2 \ll (k_1+k_2)^2 \ll M^2 \qquad . \tag{7}$$

The key observation concerns the phase space available for the evolution of the jets k_1 and k_2 in this region. The boundary on the energy fraction x_1 of jet 1 comes from[10]

i) fragmentation of jet 1: this gives $x_1 \gtrsim \sqrt{4k_1^2/M^2}$;

ii) recoil of jet 2: after using the angular ordering to approximate the phase space in Eq.(4), this gives $x_1 \gtrsim k_1^2/[M^2(1-z)]$.

For $z \to 1$ the tightest constraint is set by ii).

By evaluating the phase space in Eq. (4) explicitly we obtain[10]

$$\begin{aligned} \frac{1}{\Gamma}\frac{d\Gamma}{dz} \quad &\simeq \quad \frac{1}{16(2\pi)^3(\pi^2-9)} \\ &\times \quad \int_0^1 dx_1 \int_0^1 dx_2 \ \mathcal{M}_0(x_1,x_2,z) \ \delta(z-2+x_1+x_2) \ \Theta(x_1+x_2-1) \\ &\times \quad \int_0^\infty dk_1^2 \ J_g\left(M^2(1-z),k_1^2\right) \ \Theta(M^2x_1(1-z)-k_1^2) \ \Theta(M^2x_1^2/4-k_1^2) \\ &\times \quad \int_0^\infty dk_2^2 \ J_g\left(M^2(1-z),k_2^2\right) \ \Theta(M^2x_2(1-z)-k_2^2) \ \Theta(M^2x_2^2/4-k_2^2) \quad , \end{aligned} \tag{8}$$

where $\mathcal{M}_0$ is given in Eq. (2).

Eq. (8) allows us to discuss the logarithmic behaviors in the endpoint region by using the results for the jet mass distributions. By expanding the double logarithmic expression (6) in powers of α_s we see that the photon spectrum contains corrections involving integrals of the form

$$\int_{1-z}^1 dx_1 \ln\left(\frac{M^2(1-z)}{M^2x_1(1-z)}\right) = -\int_{1-z}^1 dx_1 \ln x_1 \qquad . \tag{9}$$

That is, logarithmic contributions in x_1 arise, which are important at the kinematic limit $x_1 \to 0$, but these never give rise to logarithms of $(1-z)$ in the spectrum. All $\ln(1-z)$ cancel because of the form of the constraint on the jet mass, which in turn is a consequence of the angular ordering of the gluon radiation.

Therefore, higher orders of perturbation theory do not contribute a Sudakov suppression of the photon spectrum in the endpoint region. They give rise to a constant shift compared to the lowest order answer.

The picture underlying this result can be understood in simple terms. In the boundary kinematics the photon recoils against two almost-collinear gluon jets. The cancellation of Sudakov corrections reflects the fact that color is neutralized already at the level of this two-jet configuration. This situation may be contrasted with the situation one encounters in the decay of an electroweak gauge boson into jets[13,14]. Here Sudakov corrections arise precisely from the presence of color charges in two-jet kinematics.

This picture also indicates that no cancellation should occur for the color-octet Fock state in the quarkonium. In this case, we expect the usual Sudakov suppression to take place near the endpoint of the photon spectrum. Then one of the consequences of the result presented here concerns the ratio of the color-octet to the color-singlet contributions. This ratio will be smaller than expected from the power counting in v and α_s obtained by truncating perturbation theory to fixed order[2], owing to the different high order behavior of the perturbation series in the two cases.

To conclude, we observe that resummation formulas of the type (4),(8) can be expanded to fixed order in α_s and matched with leading and next-to-leading[15] results to obtain improved predictions, valid over a wider range of photon energies. If these formulas are combined with models for the infrared behavior of the strong coupling[16], they can be used to model the nonperturbative shape functions[17] that parameterize power-like corrections[5,6] near the endpoint. Taking account of effects from the soft region will likely influence[4,15] the estimate of the uncertainty on the determination of α_s from quarkonia decays[5,9,18].

Acknowledgements

I thank S. Catani for collaboration on quarkonia decays and for discussion, and the organizers of the QCD session at DPF2000 for their invitation.

References

1. D.M. Photiadis, Phys. Lett. B **164**, 160 (1985).
2. I.Z. Rothstein and M.B. Wise, Phys. Lett. B **402**, 346 (1997); T. Mannel and S. Wolf, hep-ph/9701324.
3. F. Maltoni and A. Petrelli, Phys. Rev. D **59**, 074006 (1999).
4. S. Wolf, hep-ph/0010217.
5. J.H. Field, Nucl. Phys. **B**, Proc. Suppl. **54 A**, 247 (1997); hep-ph/0101158.
6. M. Consoli and J.H. Field, Phys. Rev. D **49**, 1293 (1994); J. Phys. G **23**, 41 (1997).
7. R.D. Field, Phys. Lett. B **133**, 248 (1983).
8. Mark II Coll., Phys. Rev. D **23**, 43 (1981); CLEO Coll., Phys. Rev. Lett. **56**, 1222 (1986); ARGUS Coll., Phys. Lett. B **199**, 291 (1987); Crystal Ball Coll., Phys. Lett. B **267**, 286 (1991).
9. CLEO Coll., Phys. Rev. D **55**, 5273 (1997).

10. F. Hautmann, hep-ph/0102336; hep-ph/9708496, in Proceedings of "Photon97", eds. A. Buijs and F.C. Erné (World Scientific 1998), p. 68.
11. S. Catani and F. Hautmann, Nucl. Phys. **B**, Proc. Suppl. **39 BC**, 359 (1995).
12. S.J. Brodsky, T.A. DeGrand, R.R. Horgan and D.G. Coyne, Phys. Lett. B **73**, 203 (1978); K. Koller and T. Walsh, Nucl. Phys. **B140**, 449 (1978).
13. See the review in Yu.L. Dokshitzer, V.A. Khoze, A.H. Mueller and S.I. Troyan, Rev. Mod. Phys. **60**, 373 (1988).
14. S. Catani, L. Trentadue, G. Turnock and B.R. Webber, Nucl. Phys. **B407**, 3 (1993).
15. M. Krämer, Phys. Rev. D **60**, 111503 (1999); hep-ph/9901448.
16. R.D. Dikeman, M. Shifman and N.G. Uraltsev, Int. J. Mod. Phys. **A11**, 571 (1996); Yu.L. Dokshitzer, hep-ph/9812252, in Proceedings of ICHEP98, eds. A. Astbury, D. Axen and J. Robinson (World Scientific 1999), p. 305.
17. T. Mannel and S. Recksiegel, hep-ph/0009268; A.L. Kagan and M. Neubert, Eur. Phys. J. C**7**, 5 (1999); G.P. Korchemsky and G. Sterman, Phys. Lett. B **340**, 96 (1994).
18. M. Kobel, in "QCD and Hadronic Interactions", Proceedings of the 27th Rencontres de Moriond, ed. J. Tran Thanh Van (Editions Frontieres, 1992), p. 145.